DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Remarks on Problem 179 by TSURUICHI HAYASHI, Koto Shihan Gakko, Tokyo, Japan.

The following question proposed by Dr. L. E. Dickson, The University of Chicago, remains unsolved in Vol. XII, 1905, p. 238:

Find the roots of the algebraically solvable quintic equation

$$x^5+qx^2+px+\tfrac{1}{5}\left[\frac{q^2}{p}-\frac{p^3}{5q}\right]=0.$$

I think that the coefficients of x^2 and x must be interchanged and thus the equation must become

$$x^5+px^2+qx+\tfrac{1}{5}\left[\frac{q^2}{p}-\frac{p^3}{5q}\right]=0.$$

If so, the roots are represented by

$$-\omega_\lambda \sqrt[5]{\frac{q^2}{5p}}+\omega_\lambda{}^3 \sqrt[5]{\frac{p^3}{25q}}$$

where $\lambda=1, 2, 3, 4, 5$, and $(\omega_\lambda)^5=1$.

Remark by the PROPOSER.

The problem was printed incorrectly; it should have read $x^5+px^2+qx+\ldots$, with the letters p, q in their natural order. Since p is of the third degree in the roots, and q of the fourth, the constant term is of the fifth degree, as should be the case.

Mr. T. Hayashi's solution of the corrected equation has, doubtless by an oversight in copying, the terms ω and ω^3 interchanged.

Solution. Since the terms x^4 and x^3 are lacking, the simplest expressions to assume for the roots are

$$r_\lambda=\omega_\lambda{}^3 A+\omega_\lambda B \quad (\lambda=1, \ldots, 5;\ \omega_\lambda{}^5=1).$$

Then by $\sum\limits_{\lambda=1}^{5}\omega_\lambda{}^r=0$ for $r=1, \ldots, 4$, we have for $S_t=\Sigma r_\lambda{}^t$,

$$S_1=S_2=0,\quad S_3=15AB^2,\quad S_4=20A^3B,\quad S_5=5A^5+5B^5.$$

But for $x^5+px^2+qx+r=0$,

$$S_1=S_2=0, \quad S_3=-3p, \quad S_4=-4q, \quad S_5=-5r.$$

Hence, $A=-\sqrt[5]{\dfrac{q^2}{5p}}$, $B=\sqrt[5]{\dfrac{p^3}{25q}}$, $r=\dfrac{q^2}{5p}-\dfrac{p^3}{25q}$.

276. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

If $x_1, x_2, \ldots, x_n$ be unequal, and $f(x)$ be a rational integral function of degree $\not> n-2$, then shall

$$\sum_{r=n-1}^{r=1} \frac{f(x_r)}{(x_r-x_1)\ (x_r-x_2)\ldots(x_r-x_n)}=0.$$

Solution by the PROPOSER.

The left hand side written at length is

$$\frac{f(x_1)}{(x_1-x_2)\ (x_1-x_3)\ldots(x_1-x_n)}+\frac{f(x_2)}{(x_2-x_1)\ (x_2-x_3)\ldots(x_2-x_n)}+\ldots$$

$$+\frac{f(x_n)}{(x_n-x_1)\ (x_n-x_2)\ldots(x_n-x_{n-1})}.$$

Let $\dfrac{f(x)}{(x-x_1)\ (x-x_2)\ldots(x-x_n)}\equiv\dfrac{A_1}{x-x_1}+\dfrac{A_2}{x-x_2}+\ldots+\dfrac{A_n}{x-x_n}$.

Then $A_1, A_2, A_3, \ldots, A_n$

$$=\frac{f(x_1)}{(x_1-x_2)\ (x_1-x_3)\ldots(x_1-x_n)},\quad \frac{f(x_2)}{(x_2-x_1)\ (x_2-x_3)\ldots(x_2-x_n)},\ \ldots,$$

$$\frac{f(x_n)}{(x_n-x_1)\,(x_n-x_2)\ \ldots\ (x_n-x_{n-1})}.$$

Hence, $f(x)\equiv\Sigma A_1(x-x_2)(x-x_3)\ldots(x-x_n)=$polynomial of degree $\not> n-2$.
Hence, $\Sigma A_1=0$.

This problem, as we thought, proves to be similar to Ex. 4, p. 319, 3rd Edition of Burnside and Panton's *Theory of Equations*. ED. F.

GEOMETRY.

311. Proposed by J. OWEN MAHONEY, B. E., M. Sc., Dallas High School, Dallas, Texas.

Triangle ABC is obtuse-angled at C; x, y, z are squares on the sides AC, CB, BA; LH and MJ are lines joining adjacent sides of x, z and y, z. The common chord of the circles on LH and MJ as diameters passes through C and the mid-point of HJ.